B.Sc. Part II 4th Paper

DYNAMICS

Tangential velocity $= \frac{ds}{dt} = \dot{s}$

Normal velocity $= 0$

Tangential acceleration $= \frac{d^2 s}{dt^2} = v\frac{dv}{ds}$

Normal acceleration $= \frac{v^2}{\rho}$, $\rho = \frac{ds}{d\psi}$, towards the direction of centre of curvature, ρ = radius of curvature

Angular velocity $= \frac{d\theta}{dt} = \frac{\sin\phi}{r}$

Q: One point describes the diameter OA of a circle with uniform velocity and another the semi-circumference OA from rest with uniform tangential acceleration. They start together ~~at~~ from O and arrive together at A. Show that the velocities at A are as $\pi : 1$.

Soln Let t be the time taken by the particle in arriving at A.

Let u = uniform velocity of first describing OA.

$\Rightarrow$ If $OA = 2a$ then

$OA = ut \Rightarrow 2a = ut$

$\Rightarrow t = \frac{2a}{u}$ ——— (1)

For the second point..

tangential acceleration is uniform.

$\Rightarrow$ tangential acceleration = constant

$\Rightarrow \frac{dv}{dt} = v\frac{dv}{ds}$ = constant $= k$ (say)

$\Rightarrow v\,dv = k\,ds$ Integrating, we get-

$\frac{v^2}{2} = ks + k_1$ ——— (2)

Initially when $s=0$, $v=0$, as it starts from rest.

So, eq(2) $\Rightarrow$ $0 = 0 + k_1 \Rightarrow k_1 = 0$

Putting the value of k_1 in (2), we get

$\frac{v^2}{2} = ks \Rightarrow v^2 = 2ks.$ ——— (3)

Let V = velocity when it arrives at A

Then s = semi-circumference $= \pi a$

So (3) $\Rightarrow V^2 = 2k\pi a$ ——— (4)

Taking $\frac{dv}{dt} = k \Rightarrow dv = k\,dt \Rightarrow v = kt + k_2$

when $t=0, v=0 \Rightarrow k_2 = 0 \Rightarrow v = kt$ But $t = \frac{2a}{u}$

When $t = \frac{2a}{u}$ then $v = V$

$\Rightarrow V = kt = k.\frac{2a}{u} \Rightarrow k = \frac{uV}{2a}$. so (4) $\Rightarrow$

$V^{\cancel{2}} = \cancel{2}\pi \cancel{a}.\frac{u\cancel{V}}{\cancel{2a}} \Rightarrow V = u\pi \Rightarrow \frac{V}{u} = \frac{\pi}{1}$ Proved